HURLSTONE AGRICULTURAL HIGH SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2024

HSC ASSESSMENT TASK 4
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet has been provided in the Section I booklet
- For questions in Section II-VIII, **show all relevant mathematical reasoning and/or calculations**

Total marks: 100

Section I: 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10.
  A multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Sections II – VIII: 90 marks (pages 13 -54)

- Attempt Questions 11 – 42.
  Write your solutions in the spaces provided
- There are **7 separate question/answer booklets**
  Extra working pages are available if required
- Allow about 2 hours and 45 minutes for these sections

This examination paper is not to be removed from the Examination Centre

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 HSC Mathematics Advanced Examination.

SECTION I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. For the parabola with equation $y = x^2 - 4x + 6$, which of the following is correct?

   A. has axis of symmetry $x = 2$ and vertex is at $(2, -2)$.

   B. has axis of symmetry $x = 2$ and vertex is at $(2, 2)$.

   C. has axis of symmetry $x = 6$ and vertex is at $(6, -18)$.

   D. has axis of symmetry $x = 6$ and vertex is at $(6, 18)$.

2. Consider the simultaneous equations.

$$2ax + y - 5 = 0$$
$$x + 3y - 2a = 0$$

   If the equations have no real solutions, what is the value of a?

   A. $a = \frac{1}{6}$

   B. $a = \frac{2}{3}$

   C. $a = -\frac{1}{6}$

   D. $a = -\frac{2}{3}$

Section I continued on next page ...

3. Which expression is a term of the geometric series $4x - 8x^2 + 16x^3 - \ldots$?

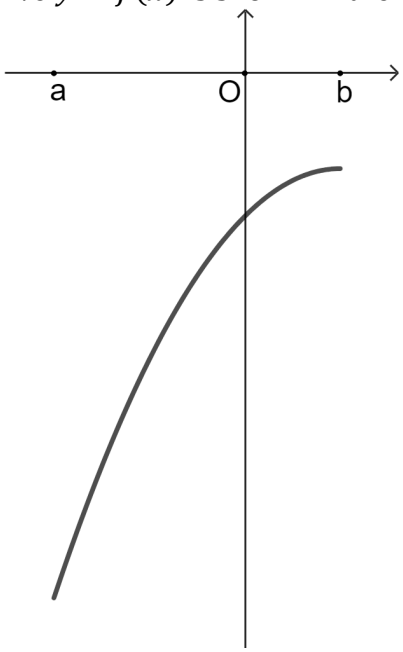
A. $-2048x^9$

B. $-2048x^{10}$

C. $2048x^9$

D. $2048x^{10}$

4. A continuous curve $y = f(x)$ is shown in the interval $a \leq x \leq b$,



Which of the following correctly describes the behaviour of $y = f(x)$ in the given domain?

A. $f(x) < 0, f'(x) < 0, f''(x) < 0$

B. $f(x) > 0, f'(x) < 0, f''(x) > 0$

C. $f(x) < 0, f'(x) > 0, f''(x) < 0$

D. $f(x) > 0, f'(x) > 0, f''(x) < 0$

Section I continued on next page ...

5. Given $f(x) = \tan^2 x$, its derivative $f'(x)$ is given by which of the following?

A. $\sec^4 x$

B. $\tan^2 x \sec^2 x$

C. $2 \tan x \sec^2 x$

D. $2 \tan^2 x \sec^2 x$

6. Which of the following is the integral of

$$\int (e^x + 2)^2 \, dx \, ?$$

A. $2e^x(e^x + 2) + c$

B. $\dfrac{(e^x + 2)^3}{3} + c$

C. $2e^{2x} + 4e^x + 4x + c$

D. $\dfrac{1}{2} e^{2x} + 4e^x + 4x + c$

7. Which of the following is NOT true?

A. $\displaystyle\int x(x^2 - 1)^2 \, dx = \frac{(x^2 - 1)^3}{6} + c$

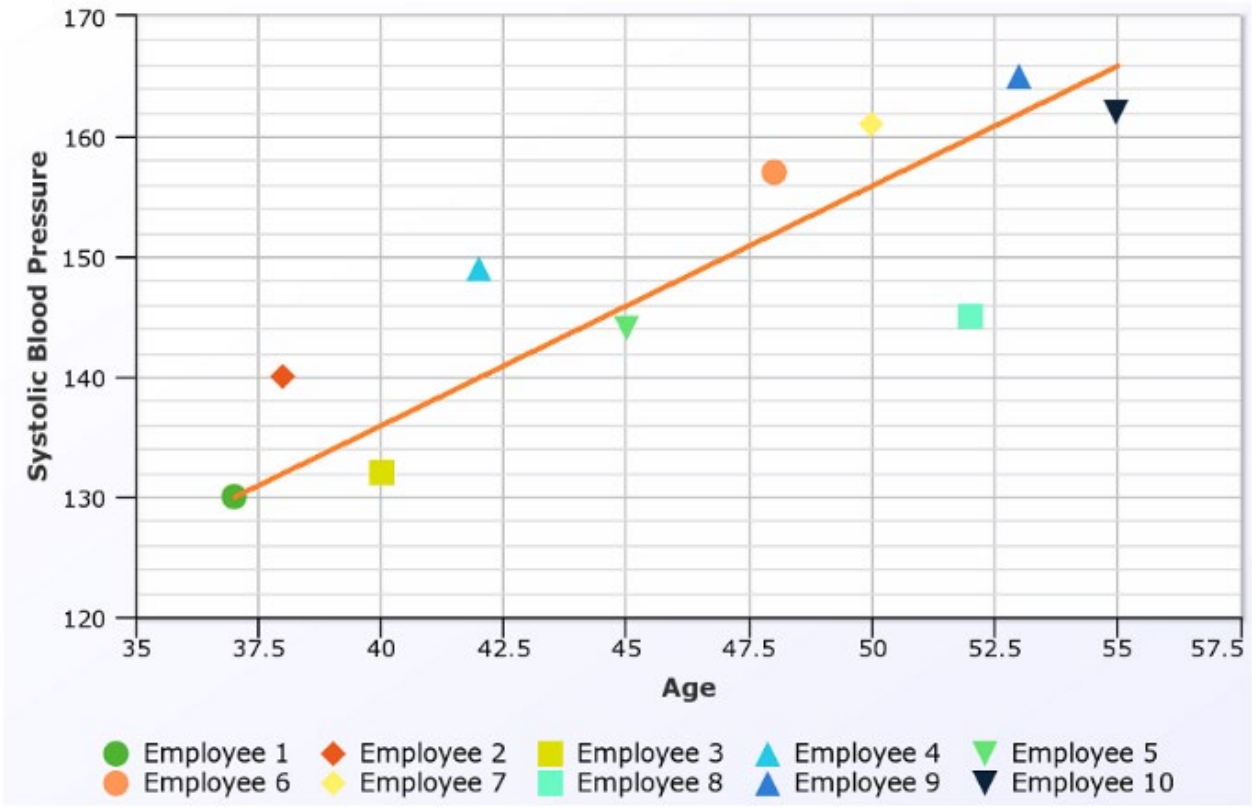
B. $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2} \, dx = \frac{9\pi}{2}$

C. $\displaystyle\int_{-1}^{1} 3^x \, dx = \frac{8}{3 \ln 3}$

D. $\displaystyle\int_{-5}^{5} 4x^4 - x^3 + \cos x \, dx = 0$

Section I continued on next page ...

8. A company records the Systolic Blood Pressure and Age for 10 employees in the following scatterplot. A correlation is noted and a line of best fit is shown.



Using "P" for blood pressure, and "A" for age, what is the equation of the line of best fit shown?

A. $P = 126 + 2A$

B. $P = 130 + 2A$

C. $P = 56 + 2A$

D. $P = 126 + \frac{A}{2}$

Section I continued on next page ...

9. Which of the following functions has a period of $\frac{\pi}{2}$?

A. $\sin\left(\frac{x}{2}-\frac{2\pi}{3}\right)$

B. $\sin\left(2x-\frac{2\pi}{3}\right)$

C. $\sin\left(4x-\frac{2\pi}{3}\right)$

D. $\sin\left(\frac{x}{4}-\frac{2\pi}{3}\right)$

10. Which of the following statements is **true** for the function

$$f(x)=\frac{2}{3}x^3-6x^2+20x-5?$$

A. $f(x)$ has a maxima.

B. $f(x)$ has a minima.

C. $f(x)$ is decreasing for all x.

D. $f(x)$ has a point of inflection.

~ End of Section I ~

SECTION II

Student Name: _____

13 marks
Attempt Questions 11 - 15
Allow about 23 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section II **Marks**

Question 11

State the domain and range of the function $y = 2\sqrt{16 - x^2}$. **2**

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Question 12

Solve the equation: **2**

$$\log_e(x + 2) = \log_e(2x)$$

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Section II continued on next page ....

Question 13

While studying a new disease, scientists found that the number of toxic cells in the bloodstream increased over time, according to the model $D(t) = 12 \times (1.16)^t$, for $t \geq 0$ and where D is the number of toxic cells in the bloodstream per litre and t is the time in hours.

(a) What is the initial number of toxic cells in the bloodstream? **1**

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(b) Determine the time it takes for the number of toxic cells in the bloodstream to first exceed 200 per litre. Give your answer to the nearest hour. **2**

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Section II continued on next page ....

Question 14

(a) Sketch the graph of $y = 2|x - 1| - 3$. Show all intercepts with the axes. **2**

(b) Hence or otherwise solve the inequation $2|x - 1| - 3 < -1$. **2**

Section II continued on next page ....

Question 15

Sketch the graph of $y = e^{(1-x)} + 2$ by transforming the graph of $y = e^x$. **2**
Show any intercepts, asymptotes and an additional point.

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~ End of Section II ~

SECTION III

Student Name: _____

10 marks
Attempt Questions 16 - 20
Allow about 17 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section III **Marks**

Question 16

Find the twentieth term in the sequence 50, 41, 32, 23, . . . **1**

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Question 17

The positive multiples of 7 are 7, 14, 21, ... **3**

Find the sum of all positive multiples of 7 less than 1000.

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Section III continued on next page ....

Question 18

Find the limiting sum of the geometric series **2**

$$2+\frac{2}{\sqrt{2}+1}+\frac{2}{\left(\sqrt{2}+1\right)^2}+\cdots$$

Leave your answer as an exact value with a rational denominator.

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Question 19

Explain why the geometric series below does NOT have a limiting sum. **1**

$$2+\frac{2}{\sqrt{2}-1}+\frac{2}{\left(\sqrt{2}-1\right)^2}+\cdots$$

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Section III continued on next page ....

SECTION IV

Student Name: _____

14 marks
Attempt Questions 21 - 25
Allow about 30 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section IV **Marks**

Question 21

Differentiate $\log_e(e^{2x}+1)$. **1**

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Question 22

For what values of x is $y = x^3 + 6x^2 - x + 4$ concave down? **2**

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Section IV continued on next page ....

Question 23

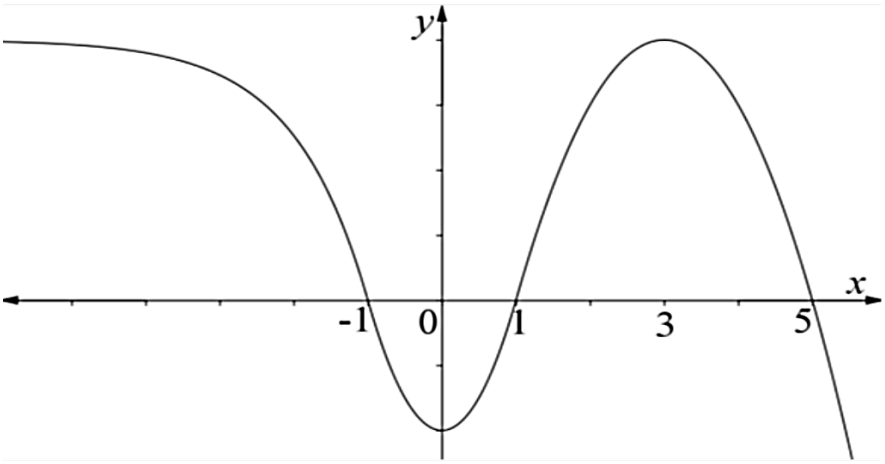
Consider the curve $y = \cos x - x$, for $0 \leq x \leq 2\pi$.

Find any stationary points and determine their nature. **3**

Section IV continued on next page ....

Question 24

The curve $y = f(x)$ shown has a maximum turning point at $x = 3$ and a minimum turning point at $x = 0$. Sketch the graph of its derivative function $y = f'(x)$. **2**



Section IV continued on next page ....

Question 25

Consider the curve $f(x) = \frac{5}{2+3^{-x}}$ for $x \geq 0$.

(a) Show that $f'(x) > 0$ for all values of x in the domain. **3**

(b) What happens to $f(x)$ as x gets larger and larger? **1**

Question 25 continued on next page ....

Question 25 continued ...

(c) Hence or otherwise, sketch the curve **2**

$f(x) = \frac{5}{2+3^{-x}}$ for $x \geq 0$, showing all important features.

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~ End of Section IV ~

SECTION V

Student Name: _____

17 marks
Attempt Questions 26 - 31
Allow about 35 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section V **Marks**

Question 26

Use the trapezoidal rule with 3 function values to approximate the value of **2**

$$\int_0^2 \sqrt{1+x^2}\, dx.$$

Give your answer correct to two decimal places.

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Question 27

If $f'(x) = 6x^2 + 3$ and $f(2) = 8$, find $f(x)$. **2**

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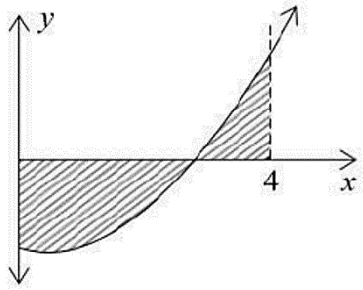
Section V continued on next page ....

Question 28

The diagram shows the graph of $y = x^2 - x - 6$. **3**

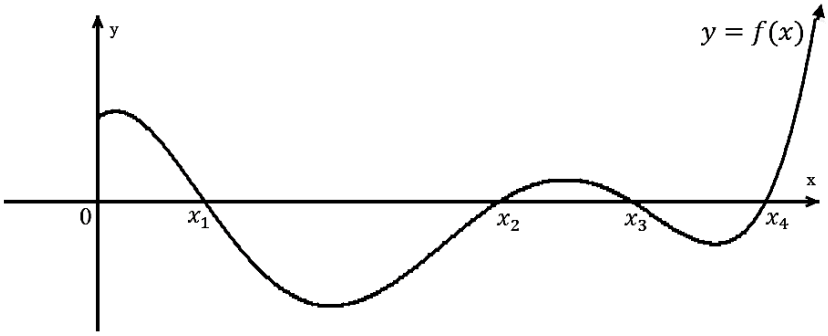


Find the area bounded by the x-axis and the graph $y = x^2 - x - 6$ between $0 \leq x \leq 4$.

Section V continued on next page ....

Question 29

The graph of the function $y = f(x)$ is shown below. **3**



It is given

$$\int_0^{x_2} f(x)\ dx = -3, \quad \int_0^{x_3} f(x)\ dx = -1 \quad \text{and} \quad \int_0^{x_4} f(x)\ dx = -4.$$

What is the value of

$$\int_{x_2}^{x_4} f(x)\ dx\ ?$$

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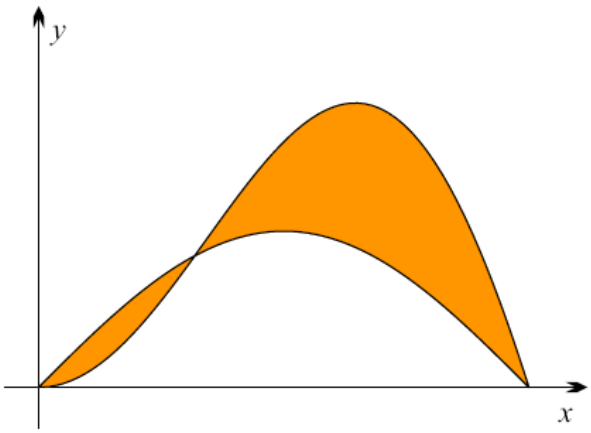
Section V continued on next page ....

Question 30

Jason is a digital designer with GraphicsAI.
He designed the logo, shown below, for CEO of QuantumVerse, Saifan.



Saifan hired James' company, The Integers, to make the logo for the entrance to his company.

James calculated that the logo is the region bounded by the curves $y = \sin x$ and $y = x \sin x$ for $0 \leq x \leq \pi$.

(a) It is given that **1**

$$\frac{d}{dx}(x \cos x) = \cos x - x \sin x \text{ . (DO NOT PROVE THIS)}$$

Show that

$$\int x \sin x \ dx = \sin x - x \cos x + C.$$

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Question 30 continued on next page ....

Marks

Question 30 continued ...

(b) Hence, calculate the area (shaded in orange) of the logo, correct to 2 decimal places. **4**

Section V continued on next page ....

Question 31

Find **2**

$$\int e^{-x} - \frac{4}{2x+1}\, dx.$$

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~ End of Section V ~

SECTION VI

Student Name: _____

12 marks
Attempt Questions 32- 36
Allow about 20 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section VI **Marks**

Question 32

A discrete random variable X has the probability distribution table shown. **3**

$X = x$	15	16	17	18
$P(x)$	0.1	m	0.25	0.4

By finding the value of m, calculate the expected value and the variance of X.

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Section VI continued on next page ....

Question 33

A group of 60 students has the following enrolment details:
26 students study Mathematics Advanced.
29 students study Biology.
14 students study both subjects.

(a) Draw a Venn Diagram to represent this situation. **1**

(b) Find the probability that a student studies Biology given that they study Mathematics Advanced. **1**

Section VI continued on next page ....

Marks

Question 34

A credit card requires a six-figure personal identification number (PIN) for purchases. The figures are chosen from the digits 0, 1, 2, 3, ..., 9.

Repetition is allowed and the PIN can start with any of the 10 digits.

The credit card is lost and the finder tries to guess the PIN by entering six digits.

(a) What is the probability that the six digits entered are the correct PIN? **1**

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(b) What is the probability that the finder will guess at least one digit in its correct position? **1**

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Section VI continued on next page ....

Marks

Question 35

The table shows the height and weight of players in a basketball team.

Height H (cm)	191	201	200	204	211	192	196	203	188	202	186
Weight W (kg)	92	99	95	109	105	97	95	100	82	103	92

(a) Using your calculator, find the correlation coefficient for this data correct to three decimal places. **1**

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(b) Describe the correlation between height and weight for this team. **1**

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Section VI continued on next page ....

Question 36

Mr Huxley recorded how many push ups he did each day for thirty days. **3**

The results are shown in this frequency table.

Push ups per day	Frequency
100	10
120	6
140	12
160	1
180	0
200	0
220	1

Using calculations, decide whether 220 push ups was an outlier.

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~ End of Section VI ~

SECTION VII

Student Name: _____

13 marks
Attempt Questions 37- 40
Allow about 23 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section VII **Marks**

Question 37

Solve $\sqrt{3}\sin 2x = -\cos 2x$ for the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. **3**

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Section VII continued on next page ....

Marks

Question 38

Prove that **3**

$$\frac{\sin x + \tan x}{1 + \sec x} = \sin x$$

Section VII continued on next page ....

Question 39

Given $y = 1 - \cos(x + \frac{\pi}{2})$,

(a) State the range of the graph using interval notation. **1**

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(b) Sketch the curve for the domain $0 \leq x \leq 2\pi$. **2**

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Section VII continued on next page ....

Question 40

Natalie and her friends have made a post-HSC adventure bucket list. One of their bucket list items is to visit the Figure Eight rock pools and a visit to Figure Eight rock pools requires extreme caution. At all times it is very important to check the depth of water before entering the pools. The depth of water at Figure Eight rock pools can be modelled by the equation

$$d = 2 + \frac{1}{2}\sin\frac{\pi}{6}t \text{, for } 0 \leq t \leq 12$$

where d is in metres and t is time after 7 am.

(a) What is the depth of the water at high tide? **1**

(b) Figure Eight rock pools is only safe to visit when the depth of the water is below 1.75 m. Between what times is it safe to visit the pools? **3**

~ End of Section VII ~

SECTION VIII

Student Name: _____

11 marks
Attempt Questions 41- 42
Allow about 17 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Trial Examination Section VIII **Marks**

Question 41

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. **3**

Let r metres be the radius of the circle and x metres be the length of each side of the square.

The combined perimeter is given as $2\pi r + 4x = 28$ and the combined area, A, is given by:

$$A = \pi\left(\frac{14 - 2x}{\pi}\right)^2 + x^2 \quad (\textbf{DO NOT PROVE THIS})$$

What should be the lengths of each piece so that the combined area of the circle and the square is minimum? Leave your answer in simplest exact form.

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Question 41 extra writing space on next page ...

Question 42

The population size P of a species of birds living at a wildlife preserve increases at a rate of

$$\frac{dP}{dt} = 9e^{\frac{t^2}{5}} - 2t \quad for\ t \geq 0,$$

where t is the time in months. It is known that the initial population of birds is 34.

(a) You are given $\int_0^4 e^{\frac{t^2}{5}}\, dt \approx 22.26$, show that $P = 218$ to the nearest integer at $t = 4$. **2**

A coal mine was recently built near the wildlife preserve and pollution from the mine affects the population of the birds from $t = 4$ onwards. Kevin who works as an environmentalist, models that the population size of the birds can then be approximated by $P = Ate^{-0.05t} - 100,\ (t \geq 4)$.

(b) Using part (a), show that $A = 97$. **1**

Question 42 continued on next page ...

Marks

Question 42 continued ...

(c) Determine the maximum population size, leave your answer as an integer. **3**

Question 42 continued on next page ...

Question 42 continued ...

(d) The population size of the birds is modelled by $P = 97te^{-0.05t} - 100$, for $t \geq 4$, as shown and derived in part (b). Make $e^{0.05t}$ the subject of this equation. **1**

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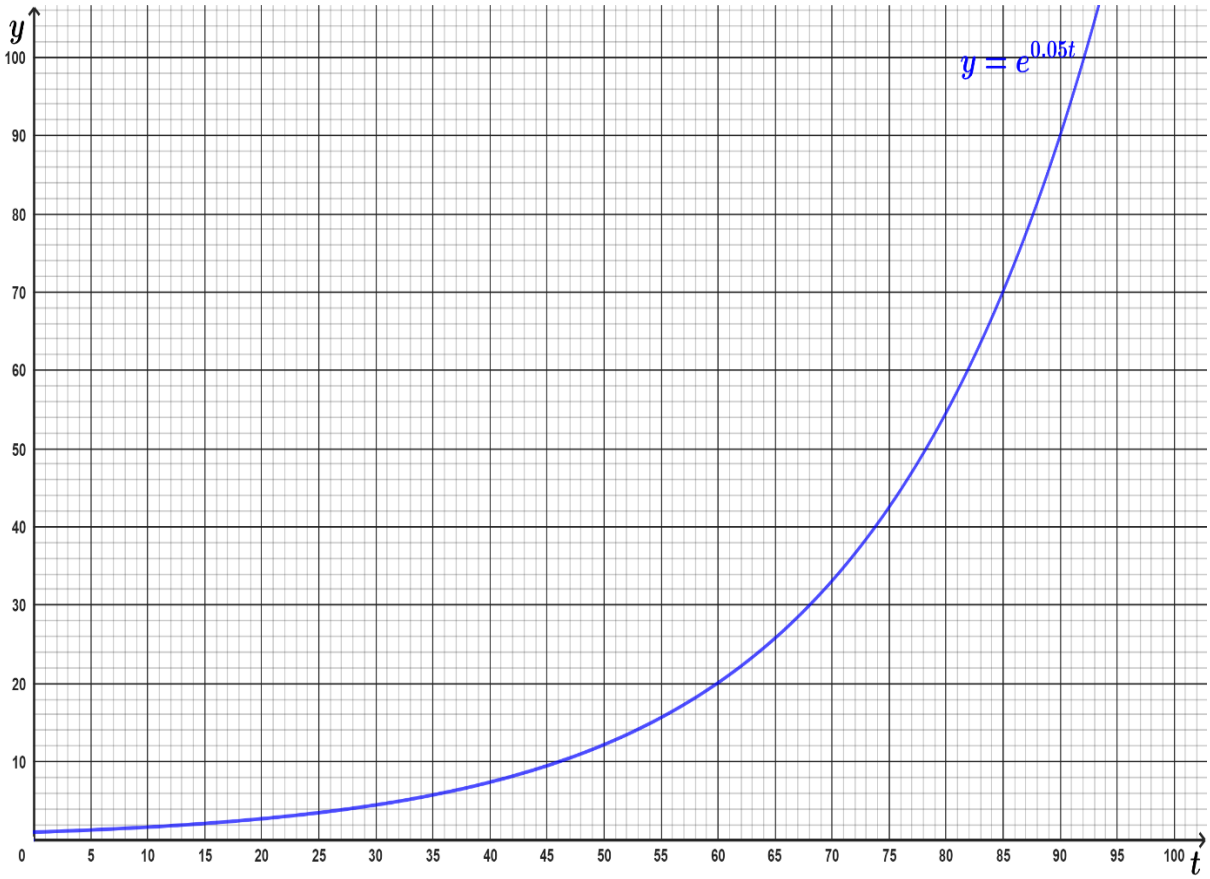
(e) Hence use the graph of $y = e^{0.05t}$ given below to find the value of t, correct to the nearest whole number for when the birds become extinct due to pollution. **1**



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~End of Section VIII ~

2024 Year 12 Mathematics Advanced Marking Guidelines

MC Q1 (MA11-2, 12-1)

$y = x^2 - 4x + 6$
Axis of symmetry: $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$
When $x = 2, y = (2)^2 - 4(2) + 6 = 2$
Therefore, vertex at $(2,2)$

$\therefore$ ***Answer: B***

MC Q2 (MA11-2, 12-1)

$2ax + y - 5 = 0 \ldots \ (1)$
$x + 3y - 2a = 0 \ldots \ (2)$
From (1) $y = -2ax + 5$
From (2) $y = -\frac{1}{3}x + \frac{2}{3}a$
Both are linear relationships.
No real solutions, hence no points of intersection. This occurs when lines are parallel.
That is, $-2a = -\frac{1}{3}$
$\therefore a = \frac{1}{6}$

$\therefore$ ***Answer: A***

MC Q3 (MA12-4)

Option A and D can be eliminated because they have the wrong sign.
Noticing the coefficients of each term are powers of 2, we get $2^{11} = 2048$. Hence, option B.

$\therefore$***Answer: B***

MC Q4 (MA12-6)

$f(x) < 0$ as curve is below the x-axis
$f'(x) > 0$ as curve is increasing (positive gradient)
$f''(x) < 0$ as curve is concave down

$\therefore$***Answer: C***

MC Q5 (MA12-6)

$f(x) = \tan^2 x = (\tan x)^2$

$f'(x) = 2\sec^2 x \tan x$ using $\frac{d}{dx}(f(x))^n = nf'(x)[f(x)]^{n-1}$
and

$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 x$

∴Answer: C

MC Q6 (MA12-7)

$\int (e^x + 2)^2 \, dx$
$= \int e^{2x} + 4e^x + 4 \; dx = \frac{1}{2}e^{2x} + 4e^x + 4x + c$

∴Answer: D

MC Q7 (MA12-7)

From reference sheet, using the integrals formulas options A and C can be eliminated as they are true.
Option B is the area of a semi circle with radius 3 which is correct.
Option D is the only one left.

∴Answer: D

MC Q8 (MA12-8)

Determining the equation of the line in the diagram:
Gradient $= \frac{166-130}{55-37} = \frac{36}{18} = 2$ Hence $2A$ is part of the solution.

The constant term in the equation is the value when $A = 0$, so 130 and 126 are not reasonable answers, because the diagram has 35 at its left, but the line would go much lower than 126 by the time A is zero.

∴Answer: C

MC Q9 (MA12-5)

For $asin(bx \pm c)$ period will equal to $\frac{2\pi}{b}$

$$\frac{2\pi}{b} = \frac{\pi}{2}$$
$$b = 4$$

∴Answer: C

MC Q10 (MA12-3)

$$f(x) = \frac{2}{3}x^3 - 6x^2 + 20x - 5$$
$$f'(x) = 2x^2 - 12x + 20$$

$$f''(x) = 4x - 12$$

$$f'(x) = 0 \quad \Rightarrow \quad 2x^2 - 12x + 20 = 0$$

$$x^2 - 6x + 10 = 0$$
$$\Delta = (-6)^2 - 4 \times 10 = -4 < 0$$

No solutions and always positive for all x as $a = 2 > 0$

$\therefore f'(x) > 0 \; for \; all \; x \Rightarrow f(x)$ does not have a maxima or a minima and is increasing for all x

$$f''(x) = 0 \quad \Rightarrow \quad 4x - 12 = 0$$
$$x = 3$$

$$f''(2) = 4(2) - 12 = -4 < 0$$
$$f''(4) = 4(4) - 12 = 4 > 0$$

Since there is a change in concavity $f(x)$ has a point of inflection.

∴Answer: D

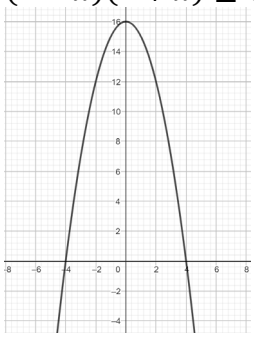
2024 Yr12 HSC Assessment Task 4 Mathematics Advanced

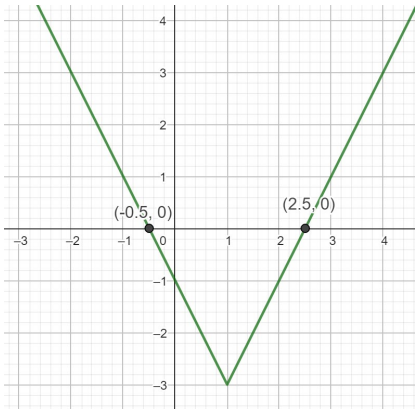
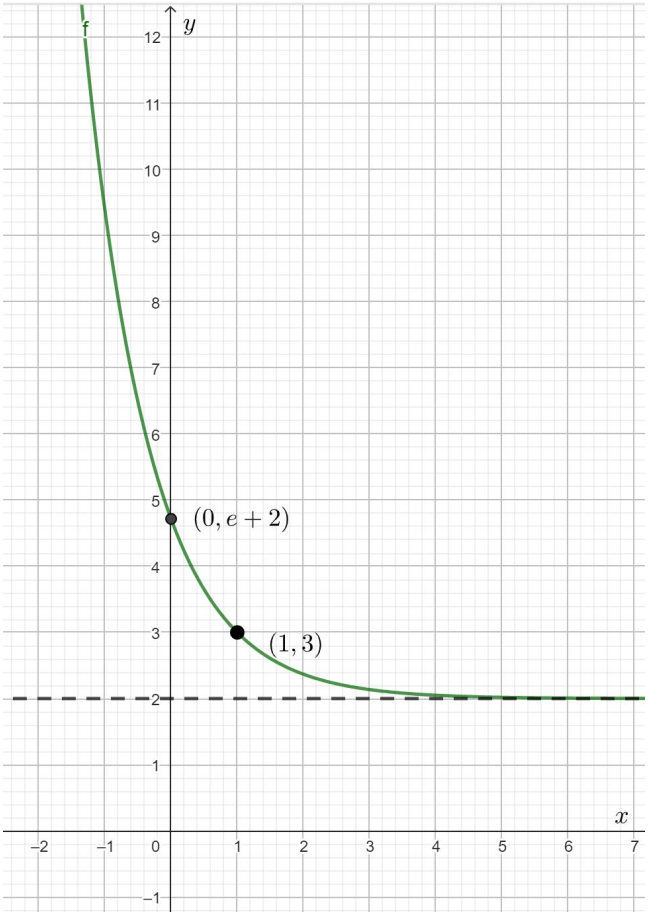
Outcomes Addressed

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-6 manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems

MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts

Outcome	Solutions	Marking Guidelines
MA11-2, 12-1	**Q11** *Domain*: $16 - x^2 \geq 0$ Range: If $x^2 = 16, \quad y = 0$ $(4 - x)(4 + x) \geq 0$ If $x^2 = 0, \quad y = 8$  $\therefore D: -4 \leq x \leq 4$ *or* $[-4,4]$ $\therefore R: \ 0 \leq y \leq 8$ *or* $[0,8]$	**2 marks** Correct domain and range **1 mark** One correct, domain or range
MA11-6, 12-1	**Q12** $x + 2 = 2x \quad \therefore x = 2$ (Since $x + 2 > 0, x > -2$ and $2x > 0, x > 0$ hence $x > -2$)	**2 marks** Correct solution **1 mark** Some progress towards correct solution
MA11-6	**Q13** **(a)** When $t = 0$, $D(0) = 12 \times 1.16^0 = 12$ toxic cells	**1 mark** Correct value
	(b) require $12 \times 1.16^t > 200$ $1.16^t > \frac{200}{12}$ $log_{10}(1.16^t) > log_{10}\left(\frac{200}{12}\right)$ $t\ log_{10}(1.16) > log_{10}\left(\frac{200}{12}\right)$ $t > \frac{log_{10}\left(\frac{200}{12}\right)}{log_{10}1.16} = 18.9557\ hours \approx 19\ hours$	**2 marks** Correct solution **1 mark** Some progress towards correct solution

MA12-1	**Q14(a)**  **(b)** Draw the line $y = -1$ on the graph above. Solution is $0 < x < 2$	**2 marks** Correct graph will *all* intercepts correctly shown **1 mark** Partially correct graph with at least one correct intercept **2 marks** Correct solution **1 mark** Some progress towards correct solution
MA12-1	**Q15**  	**2 marks** Correct graph with y–intercept, asymptote and additional point correctly shown **1 mark** Some progress towards correct graph

2024 Yr12 HSC Assessment Task 4 Mathematics Advanced

Section III Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-4 - applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems

Solutions	Marking Guidelines
16) $$T_{20} = 50 + (20-1)(-9) = -121$$	**1 Mark** Correct Solution
17) First, need to find $T_n < 1000$ $$1000 > 7 + (n-1) \times 7$$ $$n < 142.9$$ There are 142 multiples of 7 less than 1000. $$S_{142} = \frac{142}{2}(2 \times 7 + (142-1) \times 7) = 71071$$ OR, alternatively: $$\frac{1000}{7} \approx 142.9$$ Rounding down we have $n = 142$. So $$S_{142} = \frac{142}{2}(2 \times 7 + (142-1) \times 7) = 71071$$	**3 Marks** Correct Solution **2 Marks** Significant correct progress to establish sum **1 Mark** Some correct progress
18) $$r = \frac{\frac{2}{\sqrt{2}+1}}{2} = \frac{1}{\sqrt{2}+1}$$ $$S_\infty = \frac{2}{1-\frac{1}{\sqrt{2}+1}} = \frac{2}{\frac{\sqrt{2}+1-1}{\sqrt{2}+1}} = \frac{2(\sqrt{2}+1)}{\sqrt{2}} = 2+\sqrt{2}$$	**2 Marks** Correct Solution **1 Mark** Significant correct progress to establish sum

19)

$$r = \frac{\frac{2}{\sqrt{2}-1}}{2} = \frac{1}{\sqrt{2}-1} \approx 2.4$$

Since r is larger that 1, it does not have a limiting sum.

1 Mark
Correct reasoning supported by calculations as a part of the explanation

20)

The nth term of an arithmetic series is given by

$$T_n = a + (n-1) \times 2$$

The 3rd, 6th and 10th terms are

$$T_3 = a + 4$$

$$T_6 = a + 10$$

$$T_{10} = a + 18$$

Since T_3, T_6 and T_{10} are the first three terms of a geometric sequence we have a common ratio. r, between the terms.

$$\frac{T_6}{T_3} = \frac{T_{10}}{T_6}$$

$$\frac{a+10}{a+4} = \frac{a+18}{a+10}$$

$$(a+10)^2 = (a+4)(a+18)$$

$$a^2 + 20a + 100 = a^2 + 22a + 72$$

$$28 = 2a$$

$$a = 14$$

So we have

$$r = \frac{a+10}{a+4} = \frac{24}{18} = \frac{4}{3}$$

3 Marks
Correct Solution

2 Marks
Significant progress where equation to find a is correctly established

1 Mark
Some correct progress where the terms of the arithmetic sequence are established

Year 12 Mathematics Advanced Trial 2024

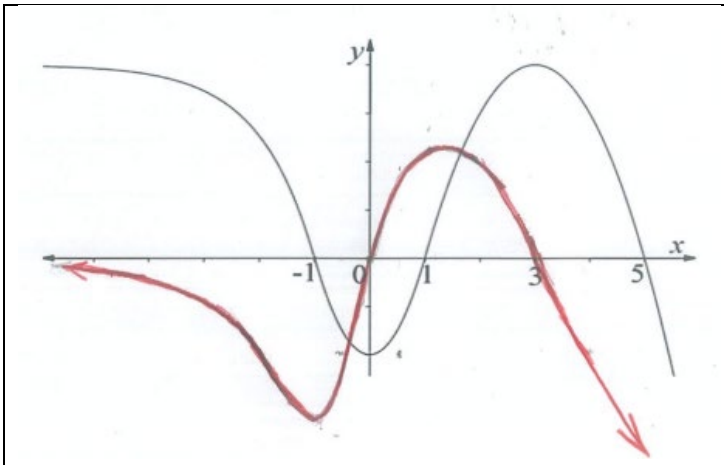
Section IV — Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-6
applies appropriate differentiation methods to solve problems

Solutions	**Marking Guidelines**
Question 21 $\frac{d}{dx}\log_e(e^{2x}+1) = \frac{2e^{2x}}{e^{2x}+1}$	1 mark: correct answer
Question 22 $y = x^3 + 6x^2 - x + 4$ $y' = 3x^2 + 12x - 1$ $y'' = 6x + 12$ Concave down when $y'' < 0$ Solving $6x + 12 < 0$ Concave down when $x < -2$	2 marks: correct solution 1 mark: correctly finds second derivative or corrects solves equivalent inequality or equivalent
Question 23 $y = \cos x - x$ $y' = -\sin x - 1$ Stationary points when $y' = 0$ Solving $-\sin x - 1 = 0$ $\sin x = -1$ $x = \frac{3\pi}{2}$ $x = \frac{3\pi}{2}$ is a horizontal point of inflection.	3 marks: correctly solves $y' = 0$, correctly tests for nature & correct nature 2 marks: two of above 1 mark: one of above
Question 24 Note: care needs to be taken when drawing the graph. It was essential to indicate that the derivative equalled zero at $x = 0$ and at $x = 3$ and was positive for $0 < x < 3$. The maximum point on the positive side of the y-axis should line up with $x = 1$ approximately where the graph of $y = f(x)$ changes from concave up to concave down. Similarly on the left of the y-axis, the minimum should occur at about $x = -1$. For $x < 0$, derivative is negative and approaches 0.	2 marks: correct solution 1 mark: substantial progress towards correct solution

x	π	$\frac{3\pi}{2}$	2π
y'	-1	0	-1



Question 25

(a) $f(x) = \frac{5}{2+3^{-x}}$

$$f'(x) = \frac{0 - 5 \times \frac{d}{dx}(2+3^{-x})}{(2+3^{-x})^2}$$

$$= \frac{-5 \times -\ln 3 \times 3^{-x}}{(2+3^{-x})^2}$$

$$= \frac{5 \ln 3 \times 3^{-x}}{(2+3^{-x})^2}$$

> 0 as $\ln 3, 3^{-x}$ & $(2 + 3^{-x})^2$ are positive.

2 marks: correctly uses quotient rule, differentiates exponential function correctly & simplifies correctly where appropriate
1 mark: uses the quotient rule correctly or correctly differentiates the exponential function plus
1 mark: demonstrates understanding by justifying terms in the numerator and denominator are positive

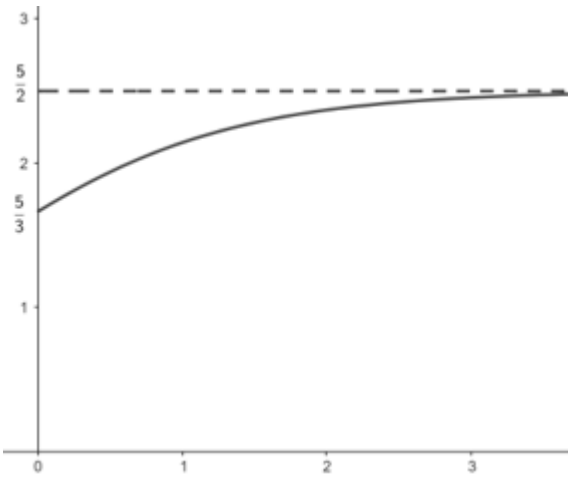
(b) As $x \to \infty, 3^{-x} = \frac{1}{3^x} \to 0$

$\therefore f(x) \to \frac{5}{2+0}$

Hence, $f(x) \to \frac{5}{2}$

1 mark: correct answer

(c)



2 marks: correct solution
1 mark: two features correct : the *y* intercept, increasing function or asymptote

Year 12	Mathematics Advanced 2024	TASK 4
Solutions and Marking Guidelines		

Outcomes Addressed in this Question

MA12-3 applies calculus techniques to model and solve problems

Solutions	Marking Guidelines

Question 26

$$\int_0^2 \sqrt{1+x^2}\,dx \approx \frac{1}{2} \times 1 \times \left(1 + 2\sqrt{2} + \sqrt{5}\right)$$

$$\approx 3.03$$

2 marks – Correct solution
1 mark – Substantially correct solution.

Question 27

$f'(x) = 6x^2 + 3$

$f(x) = 2x^3 + 3x + c$

$f(2) = 2(2)^3 + 3(2) + c = 8$

$c = -14$

$\therefore\ f(x) = 2x^3 + 3x - 14$

2 marks – Correct solution
1 mark – Substantially correct solution.

Question 28

$y = x^2 - x - 6$
$y = (x-3)(x+2)$

$0 = (x-3)(x+2)$
$x = 3$ or $x = -2$
Not a solution as $0 \le x \le 4$

$$A = \left| \int_0^3 x^2 - x - 6\ dx \right| + \int_3^4 x^2 - x - 6\ dx$$

$$= \left| \frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_0^3 \right| + \frac{x^3}{3} - \frac{x^2}{2} - 6x \Big]_3^4$$

$$= \left| -\frac{27}{2} \right| + \frac{17}{6}$$

$$= \frac{49}{3}\ \text{u}^2$$

3 marks – Correct solution
2 marks – Substantially correct solution.
1 mark – some correct working towards correct solution

Question 29

$$\int_{x_2}^{x_3} f(x)\,dx = -1 - (-3) = 2$$

$$\int_{x_3}^{x_4} f(x)\,dx = -4 - (-1) = -3$$

$$\therefore \int_{x_2}^{x_4} f(x)\,dx = 2 + (-3) = -1$$

3 marks – Correct solution
2 marks – Substantially correct solution.
1 mark – some correct working towards correct solution

Question 30 (a)

$$\int \frac{d}{dx}(x\cos x) = \int \cos x\ dx - \int x\sin x\ dx$$

$$x\cos x = \int \cos x\ dx - \int x\sin x\ dx$$

$$\int x\sin x\ dx = \int \cos x\ dx - x\cos x$$

$$\therefore \int x\sin x\ dx = \sin x - x\cos x \quad \text{as required}$$

1 mark – Correct solution

Question 30 (b)

The curves intersect when

$$\sin x = x \sin x$$
$$\sin x - x \sin x = 0$$
$$\sin x(1 - x) = 0$$
$$\sin x = 0 \quad \text{or} \quad 1 - x = 0$$
$$x = 0, \pi \quad \text{or} \quad x = 1$$

$$A_1 = \int_0^1 \sin x - x \sin x \ dx$$
$$= |-\cos x - \sin x + x \cos x|_0^1$$
$$= -\cos 1 - \sin 1 + 1 \cos 1 - (-\cos 0 - \sin 0 + 0 \cos 0)$$
$$= 1 - \sin 1$$

$$A_2 = \int_1^\pi x \sin x - \sin x \ dx$$
$$= |\sin x - x \cos x + \cos x|_1^\pi$$
$$= \sin \pi - \pi \cos \pi + \cos \pi - (\sin 1 - 1 \cos 1 + \cos 1)$$
$$= -1 + \pi - \sin 1$$

Total shaded area $= 1 - \sin 1 + (-1 + \pi - \sin 1)$
$= \pi - 2 \sin 1$
$= 1.46 \text{ u}^2$

4 marks – Correct solution
3 marks – Substantially correct solution.
2 marks – half correct solution.
1 mark – some correct working towards correct solution

Question 31

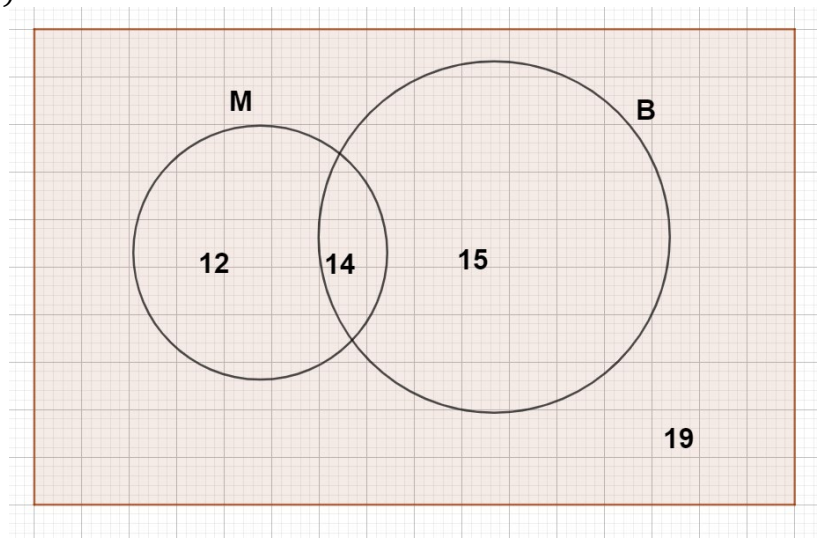
$$\int e^{-x} - \frac{4}{2x+1} \ dx$$
$$= -e^{-x} - 2 \ln|2x + 1| + c$$

2 marks – Correct solution
1 mark – Substantially correct solution.

Year 12	Mathematics Advanced Trial Examination	Assessment Task 4 2024
Section VI	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA12-8 Solves problems using appropriate statistical processes

Question	Solutions	Marking Guidelines
32	$0.1 + m + 0.25 + 0.4 = 1$ $\therefore m = 0.25$ $E(X) = 0.1 \times 15 + 0.25 \times 16 + 0.25 \times 17 + 0.4 \times 18$ $E(X) = 16.95$ $Var(X) = E((X - \mu)^2)$ $= 0.1(-1.95)^2 + 0.25(-0.95)^2 + 0.25(0.05)^2 + 0.4(0.05)^2$ OR: $Var(X) = E(X^2) - \left(E(X)\right)^2$ $= 288.35 - 287.3025$ Solution $= 1.0475$	**32** **(a)** 3 marks: Correct m, Expected value and Variance. 2 marks: 2 components correct. 1 mark: 1 component correct.
33	(a)  	**33** **(a)** 1 mark: Correct Venn diagram.
	(b) $\frac{14}{26}$ or $\frac{7}{13}$	**(b)** 1 mark: correct answer.
34	(a) P(all six correct) $= \left(\frac{1}{10}\right)^6 = \frac{1}{10^6}$ (b) $1 - P(\text{None correct}) = 1 - \left(\frac{9}{10}\right)^6 = 0.468559$	**34** **(a)** 1 mark: Correct answer or equivalent. **(b)** 1 mark: Correct numerical expression.

35	(a) By calculator in STAT mode: $r = 0.827$ (3 decimal places) (b) Since $r > 0.6$, the correlation is strong and positive.	**35** (a) 1 mark: Correct answer **(b)** 1 mark: Response that includes both features.
36	$IQR = 140 - 100 = 40$ To test for outlier for score x: $x > Q_3 + 1.5 \times IQR = 140 + 1.5 \times 40 = 200.$ $220 > 200$, therefore 220 is considered to be an outlier.	**36** 3 marks: Correct quartiles leading to IQR; Correct boundary (200); an inequality statement to show the outlier is above the boundary. 2 marks: 2 components correct. 1 mark: 1 component correct, but conclusion needed evidence.

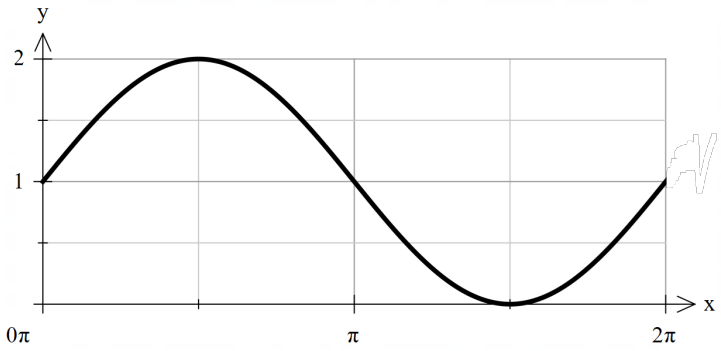
2024 Y12 Mathematics Advanced Task 4

Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-5: applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

Outcome	Solutions	Marking Guidelines
MA12-5	**Question 37** $\frac{\sin 2x}{\cos 2x} = -\frac{1}{\sqrt{3}}$ $\tan 2x = -\frac{1}{\sqrt{3}}$ $\text{Related angle} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $= \frac{\pi}{6}$ $\tan 2x$ is negative in the 2nd and 4th quadrants. Note that the new domain is $-\pi \leq 2x \leq \pi$ $\therefore 2x = \pi - \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6} = -\frac{\pi}{6}$ $2x = \frac{5\pi}{6}$ or $-\frac{\pi}{6}$ $x = \frac{5\pi}{12}$ or $-\frac{\pi}{12}$	3 marks for correct solution 1 mark for expressing the question in terms of tan 2x 1 mark for having all solutions from the correct quadrants 1 mark for having all solutions fitting the domain given
MA12-5	**Question 38** $\text{LHS} = \frac{\sin x + \tan x}{1 + \sec x}$ $= \frac{\sin x + \frac{\sin x}{\cos x}}{1 + \frac{1}{\cos x}}$ $= \frac{\sin x \cos x + \sin x}{\cos x + 1}$ $= \frac{\sin x (\cos x + 1)}{\cos x + 1}$ $= \sin x$ $= \text{RHS}$	3 marks for correct solution 1 mark for expressing the question in terms of sin x and cos x or similar merit 1 mark for multiplying by cos x / cos x or similar merit 1 mark for factorising or similar merit

MA12-5	**Question 39** a) $[0, 2]$ b)  	1 mark for correct solution 2 marks for correct solution 1 mark for correct graph shape and placement 1 mark for accurate graph details including sketching for the correct domain and axes having an even scale
MA12-5	**Question 40** a) At high tide, this is when the value of d is the largest. This occurs when $$\sin\frac{\pi}{6}t = 1$$ $$d = 2 + \frac{1}{2}(1)$$ $$= \frac{5}{2}$$ $\therefore$ The depth of water at high tide is 2.5 m. b) $$2 + \frac{1}{2}\sin\frac{\pi}{6}t < 1.75$$ $$\frac{1}{2}\sin\frac{\pi}{6}t < -0.25$$ $$\sin\frac{\pi}{6}t < -0.5$$ When $\sin\frac{\pi}{6}t = -0.5$ $$\text{Related angle} = \sin^{-1}(0.5)$$ $$= \frac{\pi}{6}$$	1 mark for correct solution 3 marks for correct solution 1 mark for making $\sin\frac{\pi}{6}t$ the subject 1 mark for solving for the values of t using trigonometric equation techniques 1 mark for determining the correct times required

	$\sin\frac{\pi}{6}t$ is negative in the 3rd and 4th quadrants. $\frac{\pi}{6}t = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$ $\frac{\pi}{6}t = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ $t = 7$ or 11 By checking its graph or by testing values, it can be determined that it is $7 < t < 11$ where $d < 1.75$. As t represents the hours after 7am, $\therefore$ It is safe to visit the pool between 2pm and 6pm.	

Year 12	Mathematics Advanced 2024	TASK 4
Solutions and Marking Guidelines		

Outcomes Addressed in this Question

MA12-3 applies calculus techniques to model and solve problems

Solutions	**Marking Guidelines**

Question 41

$$A = \pi\left(\frac{14-2x}{\pi}\right)^2 + x^2$$

$$A' = \frac{1}{\pi}2(14-2x)(-2) + 2x$$

$$A' = \frac{-8}{\pi}(7-x) + 2x$$

$$A'' = \frac{8}{\pi} + 2$$

Sub $A' = 0$ to find stationary points

$$\frac{-8}{\pi}(7-x) + 2x = 0$$

$$\frac{-56}{\pi} + \frac{8x}{\pi} + 2x = 0$$

$$\frac{-56}{\pi} + \frac{8x}{\pi} + \frac{2\pi x}{\pi} = 0$$

$$-56 + 8x + 2\pi x = 0$$

$$2x(4+\pi) = 56$$

$$x = \frac{28}{4+\pi}$$

$A'' = \frac{8}{\pi} + 2 > 0$ for all x, concave up

$\therefore x = \frac{28}{4+\pi}$ is a minimum stationary point

The combined area is minimum when the length of the two portions are :

$$4x = 4\left(\frac{28}{4+\pi}\right) = \frac{112}{4+\pi}m$$

and

$$28 - 4x = 28 - \frac{112}{4+\pi}$$

$$= \frac{28\pi}{4+\pi}m$$

3 marks – Correct solution

2 marks – Substantially correct solution.

1 mark – some correct working towards correct solution

Question 42

a)

$$P(4) - P(0) = \int_0^4 9e^{\frac{t^2}{5}} - 2t\,dt = \int_0^4 e^{\frac{t^2}{5}}\,dt - \int_0^4 2t\,dt$$

$$P(4) = 9\int_0^4 e^{\frac{t^2}{5}}\,dt - \int_0^4 2t\,dt + P(0)$$

$$= 9(22.26) - (t^2)]_0^4 + 34$$

$$= 9(22.26) - 16 + 34 = 218$$

2 marks – Correct solution
1 mark – Substantially correct solution.

b)

Use $t = 4, P = 218$

$$218 = A(4)\left(e^{-0.05\times 4} - 100\right)$$

$$A = \frac{318}{(4)(e^{-0.2})} \approx 97.10151927 \approx 97$$

1 mark – Correct solution

c) $P = 97te^{-0.05t} - 100$

$$P' = 97(1 \times e^{-0.05t} - 0.05te^{-0.05t})$$

$$= 97e^{-0.05t}(1 - 0.05t)$$

$P' = 0$ for maximum

$$97e^{-0.05t}(1 - 0.05t) = 0$$

$$e^{-0.05t}\left(1 - \frac{t}{20}\right) = 0$$

$$e^{-0.05t} \neq 0 \; and \; \left(1 - \frac{t}{20}\right) = 0$$

$\therefore t = 20$

t	19	20	21
P'	1.87569…	0	-1.6979 …
	Increasing	Stationary	Decreasing

$\therefore t = 20$ is a maximum

when $t = 20$, $\quad P = 97(20)e^{-1} - 100 \approx 613.6861159$ …

Maximum Population $= 613$

3 marks – Correct solution
2 marks – Substantially correct solution.
1 mark – some correct working towards correct solution

d) $P = 97te^{-0.05t} - 100$

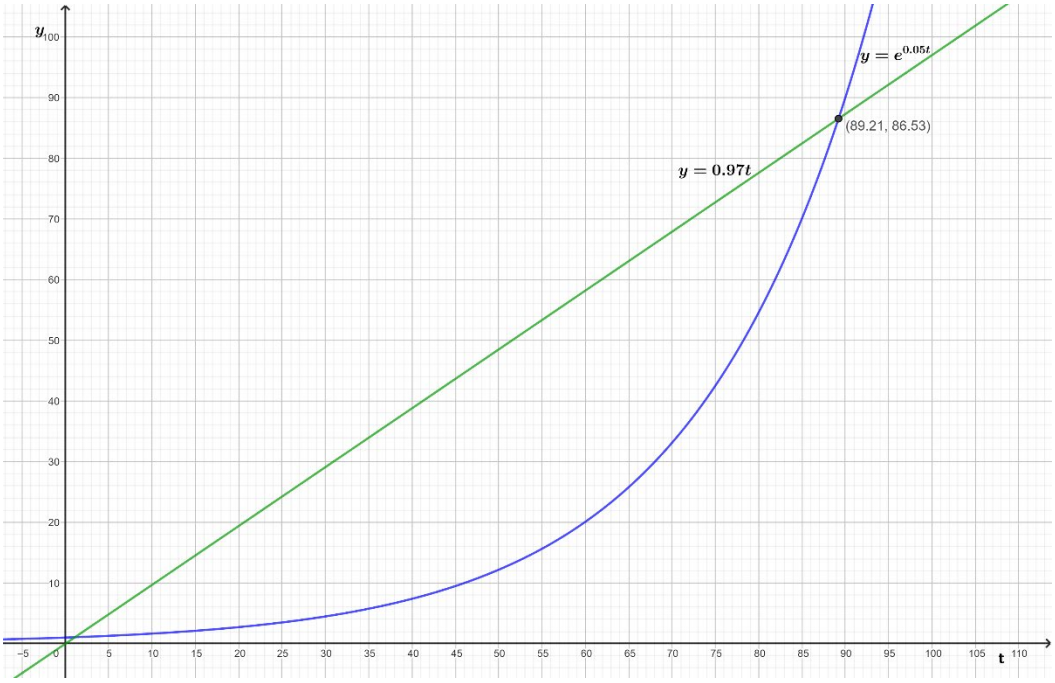
$97te^{-0.05t} = P + 100$

$e^{-0.05t} = \dfrac{P + 100}{97t}$

$e^{0.05t} = \dfrac{97t}{P + 100}$

1 mark – Correct solution

e)



1 mark – Correct solution

Birds extinct $\Rightarrow P = 0$

$e^{0.05t} = \dfrac{97t}{0 + 100}$

$e^{0.05t} = \dfrac{97t}{100} = 0.97t$

Graph $y = 0.97t$ on the graph given above.

From the graph you can conclude, it takes approximately 90 years for the birds to become extinct due to pollution.